

2.

Nr/o	Soluție	Pun- ctaj
a)	$v_{p1} = \sqrt{\frac{2RT}{M_1}} = \sqrt{\frac{2 \cdot 8,31 \cdot 300}{0,032}} = 394,7 \text{ (m/s)};$ $v_{p2} = \sqrt{\frac{2RT}{M_2}} = \sqrt{\frac{2 \cdot 8,31 \cdot 300}{0,028}} = 422 \text{ (m/s)}, \quad v_{p2} - v_{p1} = 27,3 \text{ (m/s)}$	1 p.
b)	$v_{p2} - v_{p1} = \Delta v \Rightarrow \sqrt{\frac{2RT_1}{M_2}} - \sqrt{\frac{2RT_1}{M_1}} = \Delta v \Rightarrow \sqrt{2RT_1} \left( \frac{1}{\sqrt{M_2}} - \frac{1}{\sqrt{M_1}} \right) = \Delta v \Rightarrow$ $\Rightarrow T_1 = \frac{(\Delta v)^2}{2R \left( \frac{1}{\sqrt{M_2}} - \frac{1}{\sqrt{M_1}} \right)^2} = 363,5 \text{ (K)}$	1 p.
c)	$4\pi \left( \frac{m_{01}}{2\pi kT} \right)^{3/2} v^2 e^{-\frac{m_{01}v^2}{2kT}} = 4\pi \left( \frac{m_{02}}{2\pi kT} \right)^{3/2} v^2 e^{-\frac{m_{02}v^2}{2kT}} \Rightarrow \left( \frac{m_{01}}{m_{02}} \right)^{3/2} = e^{\frac{(m_{01}-m_{02})v^2}{2kT}} \Rightarrow$ $v^2 = \frac{3kT \ln(m_{01}/m_{02})}{m_{01} - m_{02}} \Rightarrow v = \sqrt{\frac{3RT \ln(M_1/M_2)}{M_1 - M_2}} = 499,7 \text{ (m/s)} \approx 500 \text{ (m/s)}$	2 p.
d)	$N'_1/N_1 + N''_1/N_1 = 1 \quad \frac{N'_1}{N_1} = 4\pi \left( \frac{m_{01}}{2\pi kT} \right)^{3/2} \int_0^{v_{p1}} v^2 e^{-\frac{m_{01}v^2}{2kT}} dv = \frac{4}{\sqrt{\pi}} \int_0^1 u^2 e^{-u^2} du, \text{ unde } u = v/v_{p1}$ $\frac{N'_1}{N_1} = \frac{4}{\sqrt{\pi}} \int_0^1 u^2 \left( 1 - \frac{u^2}{1!} + \frac{u^4}{2!} - \frac{u^6}{3!} + \frac{u^8}{4!} - \dots \right) du = \frac{4}{\sqrt{\pi}} \int_0^1 \left( u^2 - \frac{u^4}{1} + \frac{u^6}{2} - \frac{u^8}{6} + \frac{u^{10}}{24} - \dots \right) du =$ $= \frac{4}{\sqrt{\pi}} \left( \frac{u^3}{3} - \frac{u^5}{5} + \frac{u^7}{14} - \frac{u^9}{54} + \frac{u^{11}}{264} - \dots \right) \Big _0^1 = \frac{4}{\sqrt{\pi}} \left( \frac{1}{3} - \frac{1}{5} + \frac{1}{14} - \frac{1}{54} + \frac{1}{264} - \dots \right) = 0,43, \quad \text{întrucât}$ <p>deja al 5-lea termen <math>1/264 &lt; 0,01</math>; <math>N''_1/N_1 = 1 - N'_1/N_1 = 0,57</math> ;  pentru azot părțile vor fi aceleași</p>	3 p.
e)	$\langle v \rangle = \int_{v_{p1}}^{\infty} \frac{v dN}{N''_1} = \frac{1}{0,57} \int_{v_{p1}}^{\infty} \frac{v dN}{N_1} = \frac{1}{0,57} \int_{v_{p1}}^{\infty} v dP(v) = \frac{1}{0,57} 4\pi \left( \frac{m_{01}}{2\pi kT} \right)^{3/2} \int_{v_{p1}}^{\infty} v^3 e^{-\frac{m_{01}v^2}{2kT}} dv =$ $\frac{1}{0,57} \sqrt{\frac{8kT}{\pi m_{01}}} \int_1^{\infty} x e^{-x} dx, \quad \text{unde } x = \frac{m_{01}v^2}{2kT};$ $\langle v \rangle = \frac{1}{0,57} \sqrt{\frac{8kT}{\pi m_{01}}} \frac{2}{e} = 1,29 \sqrt{\frac{8kT}{\pi m_{01}}} = 1,29 \sqrt{\frac{8RT}{\pi M_1}} = 574,6 \text{ (m/s)}$	3 p.